4.3 Review: Trigonometric Functions of Any Angle

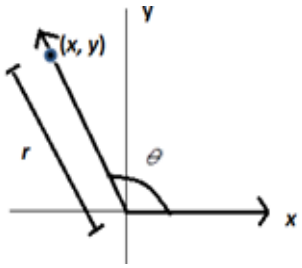
Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}, \quad x \neq 0 \qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$$

$$\sec\theta = \frac{r}{x}, \quad x \neq 0 \qquad \csc\theta = \frac{r}{y}, \quad y \neq 0$$



Example 1: Let (7, -4) be a point on the terminal side of θ. Find the sine, cosine, and tangent of θ.

Example 2: Given $\cos\theta = \frac{2}{3}$ and $\tan\theta < 0$, find $\csc\theta$.

Example 3: Evaluate the cosecant and cotangent functions at 0 and $\frac{\pi}{2}$**.**

Definition of Reference Angles

Let θ be an angle in standard position. Its **reference angle** is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

In the space below, sketch a graph of the reference angle θ **in Quadrants II, III, and IV.**

Example 4: Find the reference angle θ'**. Sketch a graph of each reference angle.**

a. $\theta = \frac{7\pi}{9}$ **b.** $\theta = 1.7$ **c.** $\theta = 144^\circ$

Evaluating Trigonometric Functions of Any Angle

To find the value of a trigonometric function of any angle θ:

1. Determine the function value of the associated reference angle θ'.
2. Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

Trigonometric Values of Common Angles

θ **(degrees)**	0°	**30°**	**45°**	**60°**	**90°**	**180°**	**270°**
θ **(radians)**							
sin θ							
cos θ							
tan θ							

Example 5: Evaluate each trigonometric function.

a. $\sin\frac{5\pi}{3}$ **b.** $\cos\left(-60^\circ\right)$ **c.** $\tan\frac{17\pi}{6}$

Example 6: Let θ be an angle in Quadrant III such that $\sin\theta = -\frac{5}{13}$. Find (a) $\sec\theta$ and (b) $\tan\theta$ by using trigonometric identities.

Example 7: Use a calculator to evaluate each trigonometric function.

a. $\cot 375^\circ$ **b.** $\sin(-4.1)$ **c.** $\sec\frac{3\pi}{8}$